

•  $(E_\theta) : z^6 - 2z^3 \cos \theta + 1 = 0$  : - (11)  
 •  $\theta \in \mathbb{R}$

**تمرين 02**

•  $z_0^{15} \quad z_0^3 \quad z_0^2 \quad z_0 = \frac{5+3\sqrt{3}i}{1-2\sqrt{3}i}$  - (1)

•  $(E_2) : z^2 - 4\bar{z} + 4 = 0$  - (2)  
 •  $(E_1) : \bar{z} + |z| = 6 + 2i$

•  $(E_3) : iz^2 - 2\bar{z} + 2 - i = 0$   
 •  $z_1^2 \quad z_1 = \sqrt{2-\sqrt{3}} - i \sqrt{2+\sqrt{3}}$  - (3)

(P) •  $z' = \frac{z+2}{z-2i}$  :  $\mathbb{C} - \{2i\}$  - (4)

•  $(\Gamma_2) = \{M(z) \in (P) / z' \in \mathbb{R}\}$   $(\Gamma_1) = \{M(z) \in (P) / |z'| = 1\}$   
 •  $(\Gamma_4) = \{M(z) \in (P) / \arg(z') \equiv \pi [2\pi]\}$   $(\Gamma_3) = \{M(z) \in (P) / z' \in i\mathbb{R}^*\}$   
 •  $(\Sigma_1) \quad (P)$  - (5)

•  $(\Sigma_1) = \{M(z) \in (P) / z + \bar{z} + z\bar{z} = 0\}$   
 C  $z_C \quad B(\sqrt{3}) \quad A(i) \quad (P)$  - (6)

•  $(P) \quad M(z) \quad (\Sigma_2)$  - (7)

•  $P(-3i) \quad N(i\bar{z}) \quad M(z)$   
 •  $(E_1) : z^8 \bar{z}^3 - 1 = 0$  :  $\mathbb{C}$  - (8)

$(E_2) : z^2 - 2z + 1 + \cos 2\alpha - i \sin \alpha = 0$  : - (9)  
 •  $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

**تمرين 01**

•  $z_3 = \frac{3+4i}{(2+3i)(4+i)} \quad z_2 = \frac{(2+i)(3+2i)}{2-i} \quad z_1 = \frac{2-\sqrt{3}i}{\sqrt{3}-2i}$  - (1)

•  $z_3 = (-1+\sqrt{3}i)^5 \quad z_3 = -1+\sqrt{3}i \quad z_2 = -\sqrt{3}+i \quad z_1 = -2+2i$  - (2)

•  $\left] -\frac{\pi}{2}, \frac{\pi}{2} \right[ \cup \left] \frac{\pi}{2}, \frac{3\pi}{2} \right[ \quad \alpha$  - (3)

•  $z = 1+i \tan \alpha$  - (4)

$z_3 = \sin \alpha - i \cos \alpha \quad z_2 = -\sin \alpha + i \cos \alpha \quad z_1 = \sin \alpha + i \cos \alpha$   
 $z_6 = 1 - \cos \alpha + i \sin \alpha \quad z_5 = 1 + \cos \alpha + i \sin \alpha \quad z_4 = -\sin \alpha - i \cos \alpha$

•  $\alpha \in \mathbb{R} \quad z_8 = 1 - \sin \alpha + i \cos \alpha \quad z_7 = 1 + \sin \alpha + i \cos \alpha$  - (5)

•  $(E) : z^2 + (-3+i)z + 4 - 3i = 0$   
 $(E') : z^2 + (1-5i)z - 3i - 6 = 0$  :  $\mathbb{C}$  - (6)

•  $(E'') : z^3 - (6+5i)z^2 + (7+17i)z + 2 - 14i = 0$  - (7)

•  $P_3(x) = \sin x \cdot \cos^4 x \quad P_2(x) = \sin^4 x + 3 \cos^4 x \quad P_1(x) = \cos^5 x$  - (8)

•  $z_2 = \left( \frac{1+\sqrt{2}+i}{1+\sqrt{2}-i} \right)^{20} \quad z_1 = \left( \frac{1+i\sqrt{3}}{1-i} \right)^{20}$  - (9)

•  $S = \cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{9\pi}{11}$  - (10)

$(E') : \left(\frac{2z+1}{z-i}\right)^4 = 1 : \quad \mathbb{C} \quad \text{-}(2)$ <p style="text-align: center;">(C) (P)</p>
<p style="text-align: right;"><b>تمرين 08</b> •</p> <p style="text-align: center;">: <math>\mathbb{N}^* \quad n \quad \mathbb{R} - \{2k\pi/k \in \mathbb{Z}\} \quad x</math></p> <p><math>Y_n = \sin x + \sin 2x + \dots + \sin nx \quad X_n = 1 + \cos x + \cos 2x + \dots + \cos nx</math></p> <p style="text-align: center;">: <math>X_n + iY_n = \frac{e^{i(n+1)x} - 1}{e^{ix} - 1} : \quad \mathbb{N}^* \quad n</math></p> <p style="text-align: center;">. <math>Y_n = \sin\left(\frac{nx}{2}\right) \cdot \frac{\sin\left(\frac{(n+1)x}{2}\right)}{\sin\left(\frac{x}{2}\right)} \quad X_n = \cos\left(\frac{nx}{2}\right) \cdot \frac{\sin\left(\frac{(n+1)x}{2}\right)}{\sin\left(\frac{x}{2}\right)}</math></p>
<p style="text-align: right;"><b>تمرين 09</b> •</p> <p style="text-align: center;">(S<sub>n</sub>)<sub>n ∈ ℕ*</sub> <math>z = \cos \frac{\pi}{n} + i \sin \frac{\pi}{n} : \quad \mathbb{N}^* \quad n</math></p> <p style="text-align: center;">. <math>S_n = \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{(n-1)\pi}{n} :</math></p> <p style="text-align: center;">. <math>\forall n \in \mathbb{N}^* : 1 + z + z^2 + \dots + z^{n-1} = i \cdot \frac{\sin \frac{\pi}{n}}{1 - \cos \frac{\pi}{n}} : \quad \text{-}(1)</math></p> <p style="text-align: center;">. <math>\left(\frac{S_n}{n}\right)_{n \in \mathbb{N}^*} \quad \forall n \in \mathbb{N}^* : S_n = \frac{1}{\tan\left(\frac{\pi}{2n}\right)} : \quad \text{-}(2)</math></p>
<p style="text-align: right;"><b>تمرين 10</b> •</p> <p style="text-align: center;">: <math>\mathbb{C}</math></p> <p style="text-align: center;">. <math>P(z) = z^3 - 2(\sqrt{3} + i)z^2 + 4(1 + \sqrt{3}i)z - 8i</math></p> <p style="text-align: center;">. <math>z_0 \quad (E) : P(z) = 0 \quad \text{-}(1)</math></p>

<p style="text-align: right;"><b>تمرين 03</b> •</p> <p style="text-align: center;">(E) : <math>z^2 - mz + m(1+i) - 2i = 0 : \quad \mathbb{C}</math></p> <p style="text-align: center;">. <math>m \in \mathbb{C}</math></p> <p style="text-align: center;">. (E) <math>S_m \quad \text{-}(1)</math></p> <p style="text-align: center;">. (E) <math>m \quad \text{-}(2)</math></p>
<p style="text-align: right;"><b>تمرين 04</b> •</p> <p style="text-align: center;">: <math>\mathbb{C} \quad \mathbb{R}^* \times \mathbb{R} \quad (\lambda, \theta)</math></p> <p style="text-align: center;">. (E') : <math>z^2 - 2(\lambda \cos \theta + i \sin \theta)z + \lambda^2 - 1 = 0</math></p> <p style="text-align: center;">. <math>\mathbb{R}^* \times \mathbb{R} \quad (\lambda, \theta) \quad z_2 \quad z_1 \quad (E') \quad \text{-}(1)</math></p> <p style="text-align: center;">. (E') <math>z_2 \quad z_1 \quad \theta \quad \lambda \quad \text{-}(2)</math></p>
<p style="text-align: right;"><b>تمرين 05</b> •</p> <p style="text-align: center;">. <math>\omega = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5} : \quad \omega</math></p> <p style="text-align: center;">: <math>(2) : \omega^5 - 1 = 0 \quad (1) : \cos \frac{2\pi}{5} = \frac{1}{2} \left(\omega + \frac{1}{\omega}\right) : \quad \text{-}(1)</math></p> <p style="text-align: center;">. <math>(3) : 1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0</math></p> <p style="text-align: center;">. <math>\cos \frac{2\pi}{5} \quad (4) : X^2 + X - 1 = 0 : \quad X = \omega + \frac{1}{\omega} \quad \text{-}(2)</math></p>
<p style="text-align: right;"><b>تمرين 06</b> •</p> <p style="text-align: center;">. <math>z_0 = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7} \quad T = z_0^3 + z_0^5 + z_0^6 \quad S = z_0 + z_0^2 + z_0^4 :</math></p> <p style="text-align: center;">. <math>\text{Im}(S) &gt; 0 \quad T \quad S \quad \text{-}(1)</math></p> <p style="text-align: center;">. <math>T \quad S \quad ST \quad S+T \quad \text{-}(2)</math></p>
<p style="text-align: right;"><b>تمرين 07</b> •</p> <p style="text-align: center;">(E<sub>2</sub>) : <math>z^4 + 1 = 0 \quad (E_1) : z^4 - 1 = 0 : \quad \mathbb{C} \quad \text{-}(1)</math></p> <p style="text-align: center;">. (E) : <math>1 + z + z^2 + \dots + z^6 + z^7 = 0 :</math></p>

• **تمرین 11:**

(2)  $(E_m)$  -ب  
 $(O, \vec{e}_1, \vec{e}_2)$  (P) - (2)

$N(-i-m)$   $M(-i+m)$   $B(-i)$   $A(z_0)$   
 •  $[MN]$   $MN$  -أ  
 •  $N$   $M$   $m$  -ب  
 $R$   $\Omega$  (C)  
 $m$   $O$   $AMN$  -ج  
 $A$   $AMN$

• **تمرین 14:**

(1)  $(E): z^2 - 2iz - 2 = 0 :$   $\mathbb{C}$  - (1)  
 •  $(E): z^2 - 2ze^{i\theta} + e^{2i\theta} - 1 = 0 :$   $]0, \pi[$   $\theta$  - (2)  
 •  $(E)$   $\mathbb{C}$   
 $(O, \vec{e}_1, \vec{e}_2)$  (P) - (3)

$z_3 = -1 + e^{i\theta}$   $z_2 = 2e^{i\theta}$   $z_1 = 1 + e^{i\theta} :$   $C(z_3)$   $B(z_2)$   $A(z_1)$   
 •  $OABC$   $z_3$   $z_1$  -أ  
 $OABC$   $]0, \pi[$   $\theta$  -ب

• **تمرین 15:**

$(E_\theta): z^3 + 2(1 - \cos\theta)z^2 + (1 - 4\cos\theta)z + 2 = 0 :$   $\mathbb{C}$   
 •  $0 < \theta < \pi$   
 $\theta$   $z_0$   $(E_\theta)$  - (1)  
 $\text{Im}(z_1) < 0$   $z_2$   $z_1$   $(E_\theta)$  - (2)  
 $(O, \vec{e}_1, \vec{e}_2)$  (P) - (3)

$C(z_2)$   $B(z_1)$   $A(z_0)$   
 •  $ABCD$   $D$   $z_D$  -أ  
 $ABCD$   $]0, \pi[$   $\theta$  -ب

(2)  $P(z) = (z - z_0)(z^2 + az + b) :$   $\mathbb{R}$   $b$   $a$  - (2)  
 •  $\text{Im}(z_2) < 0$   $(E') : z^2 - 2\sqrt{3}z + 4 = 0 :$   $z_2$   $z_1$  - (3)  
 $(O, \vec{e}_1, \vec{e}_2)$  (P) - (4)  
 $C(z_2)$   $B(z_1)$   $A(z_0)$   
 $z_2 - z_0$   $z_1 - z_0$   $z_2$   $z_1$   $z_0$   
 $OABC$

• **تمرین 11:**

$P(z) = z^4 - 3z^3 + \frac{9}{2}z^2 - 3z + 1 :$   $\mathbb{C}$   
 •  $\mathbb{C}$   $z$   $\overline{P(z)} = P(\overline{z}) :$   $P(1+i)$  - (1)  
 $\frac{1}{z_0}$   $\overline{z_0}$   $P(z)$   $z_0$  - (2)  
 $P(z) = 0 :$   $\mathbb{C}$  - (3)

• **تمرین 12:**

$(E): z^3 + 2(1-i)z^2 + (1+m^2-4i)z - 2i(1+m^2) = 0$   
 $m$   $z_0$   $(E)$  - (1)  
 $(O, \vec{e}_1, \vec{e}_2)$  (P) - (2)  
 $D(-1+im)$   $C(-1-im)$   $B(-2-2i)$   $A(2i)$   
 $\mathbb{R}$   $m$   $ABCD$  -أ  
 $ABCD$   $m$  -ب

• **تمرین 13:**

$(E_m): z^3 + (3-m^2)z + 2i(1+m^2) = 0 :$   $\mathbb{C}$   
 $2$   $m$   
 $m$   $z_0$   $(E_m)$  -أ - (1)

(P) - (3)

$(O, \vec{e}_1, \vec{e}_2)$

•  $M''(z'')$   $M'(z')$   $M(z'+z'')$

•  $OM' \parallel MM''$   $\vec{OM}' \perp \vec{OM}''$  :

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• **تمرین 16:**

•  $z \in \mathbb{C} \quad \varphi \in ]-\pi, \pi]$  - (1)

•  $z = \frac{1}{2} [\sin \varphi + i(1 - \cos \varphi)]$

$\varphi \in ]0, \pi[$  - (2)

•  $z'' = \frac{z}{z-i} \quad z' = z - i$

(P) - (3)

•  $N(z'')$   $M(z')$

$N(z'')$   $M(z')$   $(\Gamma_2)$   $(\Gamma_1)$

•  $]\pi, 2\pi[$   $\varphi$

• **تمرین 17:**

•  $\alpha \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[$

•  $(E): (1+iz)^3(1-i \tan \alpha) - (1-iz)^3(1+i \tan \alpha)$

•  $z \in \mathbb{R} \quad |z+i| = |z-i|$  :  $(E)$   $z$  - (1)

•  $e^{i\alpha} = \frac{1+i \tan \alpha}{1-i \tan \alpha}$  - (2)

•  $-\frac{\pi}{2} < \varphi < \frac{\pi}{2} \quad z = \tan \varphi$  :  $\mathbb{R} \quad z$  - (3)

$(E)$   $\varphi$   $(E')$

•  $(E)$

• **تمرین 18:**

•  $\sqrt{2} \quad m \in \mathbb{C} \quad (E): mz^2 - 2z + \bar{m} = 0$  :  $\mathbb{C}$  - (1)

•  $(E) \quad \alpha \in \mathbb{R} \quad m = \sqrt{2}e^{i\alpha}$  - (2)

•  $z'' = e^{-i(\frac{\pi}{4}+\alpha)} \quad z' = e^{i(\frac{\pi}{4}-\alpha)}$